A Comparison of Theoretical Predictions with Published Experimental Measurements on the Blown Film Process

The equations governing the shape and velocity of the polymer melt in the bubble blowing part of the blown film process are derived for non-isothermal Newtonian and isothermal elastic material behavior. Measured bubble shapes (Ast, 1973; Farber, 1973) can be reproduced quite well, lying between elastic and viscous predictions. Velocity and strain rate predictions appear more sensitive to details of temperature profile and melt rheology. The results also include bubble pressure predictions or equivalently apparent viscosities for bubble blowing. The main lines of development suggested are the modeling of heat transfer and the use of a viscoelastic constitutive equation.

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SCOPE

The manufacture of plastic sheet and film, in particular from low density polyethylene, by the process of film-blowing is carried out on a large enough scale to make a theoretical study seem worthwhile. One long-term aim of this study is to predict the properties of the film produced from the design and operating variables (for example, die size, mass flow rate, blow ratio, cooling airflow) and from properties of the polymer used. However, the process is of such complexity that as a first step it is hoped to develop a mathematical model of the kinematics and dynamics of the tubular bubble of molten polymer between the annular die, where the melt is extruded and the frost line (or freeze line) where the melt solidifies and above which it does not deform appreciably. The first stage in this work dealt with the case of an isothermal Newtonian melt (Pearson and Petrie, 1970c) and the results obtained then were qualitatively reasonable, but quantitatively unlikely to be correct.

Since measurements of bubble shapes, temperature and velocity or strain rate profiles were obtained by Ast (1973) and by Farber (1973), the model has been developed to take account of the cooling of the melt between die and frost line. In addition, the effects of the weight and inertia of the melt have been allowed for. The influence of the melt's rheological properties has been studied by considering a material with a purely elastic response to stress, this being at the other extreme from the purely viscous (Newtonian) material considered earlier. The response of real polymer melts lies somewhere between these extremes, so the hope is that comparison of both calculations will give an idea of how good the predictions are.

CONCLUSIONS AND SIGNIFICANCE

As had been suspected, the isothermal Newtonian model predicted bubble shapes which were generally of an extreme form (with a long neck and very sudden blow-up) and not in quantitative agreement with experimental measurement. However, by taking the measured temperature profile and using a standard viscosity-temperature relation it has been possible to get quite close agreement between theoretical bubble shapes and experimental measurements. The effect of gravity is shown to be small for the small-scale experiments reported, but can be expected to have a larger effect on larger scale units. The effect of inertia appears to be negligible for all practical purposes—although the effort of allowing for it in the computer program is very small also.

More interestingly, the results for the elastic model (where they have been obtained) are almost as good, and in fact the measured bubble shapes lie between the elastic and viscous predictions. Predictions of velocity or strain rate profiles are in less good agreement with experiment, and we note that here the effect of gravity is more important, and also the goodness of the approximation to the temperature profile seems more important than it was in predicting bubble shape alone. Predictions of viscosity from bubble pressure or vice versa are as good as might be expected in view of the viscoelastic nature of the melt.

As a predictive tool, the model has one major defect; we need to supply the bubble temperature profile and frost line height in order to get bubble shape predictions. An important step forward will be to implement suggestions made by the author (Petrie, 1974) for including heat transfer in the model. A further step here is to consider details of air ring design. Since these are commonly concealed behind the phrase *commercial secrecy* the academic deduces that they are extremely important. Finally, in order to improve on bubble shape and velocity profile predictions, we should have a better rheological model for the melt, particularly as it solidifies and crystallizes. The complexity of the overall model is such that integral constitutive equations are unlikely to be usable and even a Maxwell model (Petrie, 1973) presents considerable difficulties.

Finally we should pause to review our goals in undertaking the mathematical modeling. We are not really interested in predicting bubble shapes and velocities accurately, except as an intermediate step, which seems essential, in the prediction of the optical and mechanical properties of the film we produce. These properties are, we expect, determined by the cooling and deformation history of the melt through more fundamental properties like orientation and crystallinity of the film. The model can supply these histories—specifically we can readily obtain deformations, deformation rates, or stresses in both machine and transverse directions as functions of distance or time (or even temperature, Farber, 1973).

The other goal towards which our model is a necessary first step is in predicting the limits for successful running of the process. Yeow (1972) has performed a linearized stability analysis, based on the isothermal model (Pearson and Petrie, 1970c), and this clearly indicates one possible, though at present somewhat daunting, way of proceeding. Whether a simpler approach is feasible or not, the question Yeow's analysis addresses must be answered, since problems of stability are of major importance to process operators.

It is to be hoped finally that, even in its present imperfect state, the model can provide some insight into the mechanics of bubble blowing and may serve as an adjunct to the engineer's practical experience when process modifications and innovations have to be evaluated.

MATHEMATICAL MODEL—THE DYNAMICS OF FILM BLOWING

In considering the flow of the molten polymer between the die and the freeze line, our principal assumptions are that the film is thin and that the flow is steady and axisymmetric. The thin film approximation has two important aspects; it allows us to treat the film as locally plane (since the principal radii of curvature are much larger than the film thickness) and it allows us to assume that the flow variables (velocity, strain rates, temperature) are constant across the film. The approximations are discussed and formally justified elsewhere (Pearson and Petrie, 1970a).

The analysis has two stages; first we consider the local equilibrium between the forces acting on the film and the stresses within it, where the principal stresses are in the machine or longitudinal direction (L), in the transverse or circumferential direction (C), and normal to the film. In the second stage, we shall consider specific constitutive equations in order to relate the stresses L and C to the strains or strain rates in the film.

The stress equilibrium equations we use are essentially those used for membrane theory, one a balance of forces over the whole cross section in the machine direction and the other a local balance between the forces in the film and the pressure difference across it. Neglecting, for the moment, gravity, inertia, surface tension, and any forces due to the cooling air we obtain (Pearson, 1966; Pearson and Petrie, 1970b, 1970c)

$$2\pi a h L \cos\theta - \pi a^2 \Delta = F \tag{1}$$

where F is constant, and

$$\Delta = \frac{hC}{a\sec\theta} - \frac{hL}{\sec\theta}\frac{d\theta}{dz} \tag{2}$$

where θ is given by

$$\tan\theta = \frac{da}{dz}, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \tag{3}$$

Surface tension is easily included by adding a term 2Γ to hC and to hL in these two equations, and the effect of this has been discussed (Petrie, 1972). If we wish to consider gravity and inertia, we must replace (1) and (2) by

$$2\pi a h L\cos\theta - \pi a^2\Delta - Mu\cos\theta = F \tag{4}$$

and

$$\Delta = \frac{hC}{a\sec\theta} - \frac{hL}{\sec\theta}\frac{d\theta}{dz} - \rho g h\sin\theta + \frac{\rho u^2 h}{\sec\theta}\frac{d\theta}{dz} \tag{5}$$

where now F is a function of z, satisfying

$$\frac{dF}{dz} = 2\pi\rho g a h \sec\theta \tag{6}$$

for a unit with the nip rolls vertically above the die. [Clearly for downward blowing all we do is replace g by $-g$ in Equations (5) and (6)]. The mass flow rate M is a constant satisfying

$$M = 2\pi\rho a h u \tag{7}$$

and the density ρ is a known function of temperature.

The arbitrary hydrostatic pressure (since we are considering an incompressible material) is determined by equating the third principle stress (in the direction normal to the film) to the atmospheric pressure which we take as zero. The fact that there is a pressure difference Δ across the film does not imply any inconsistency in our approximations, since Δ is small compared to the stresses in the film to the same extent that the film thickness h is small compared to the bubble radius a. (In all the equations above, Δ is equated to terms involving $h \cdot C$ and $h \cdot L$).

Nonisothermal Newtonian Film

The principal rates of strain at any point in the film are, using the approximation of a locally plane (unequal biaxial) elongational flow,

$$\cos\theta\frac{du}{dz}, \quad \frac{u\cos\theta}{a}\frac{da}{dz}, \quad \frac{u\cos\theta}{h}\frac{dh}{dz} \tag{8}$$

Then for a Newtonian liquid the principal extra-stresses are 2μ times these rates of strain, and eliminating the hydrostatic pressure gives the two nonzero principal stresses

$$L = 2\mu\cos\theta\left(\frac{du}{dz} - \frac{u}{h}\frac{dh}{dz}\right) \tag{9}$$

$$C = 2\mu\cos\theta\left(\frac{u}{a}\frac{da}{dz} - \frac{u}{h}\frac{dh}{dz}\right) \tag{10}$$

We substitute these in Equations (1) and (2) and then eliminate h using Equation (7), to obtain, after further rearrangement, the differential equations

$$\frac{2M\mu}{\rho}\frac{du}{dz} = -\frac{M\mu u}{\rho}\left(\frac{1}{a}\frac{da}{dz} + \frac{1}{\rho}\frac{d\rho}{dT}\frac{dT}{dz}\right) + \frac{u}{2}(F + \pi a^2\Delta)\sec^2\theta \tag{11}$$

and

$$2a(F + \pi a^2\Delta)\sec^2\theta\frac{d\theta}{dz} = \frac{2M\mu}{\rho}\left(\frac{3}{a}\frac{da}{dz} + \frac{1}{\rho}\frac{d\rho}{dT}\frac{dT}{dz}\right) + (F - 3\pi a^2\Delta)\sec^2\theta \tag{12}$$

With Equation (3) this gives us a third-order system of differential equations for the unknown functions a, u, (and θ) which we can solve if T is a known function of z, and μ and ρ are known functions of T.

The appropriate boundary conditions (Pearson and Petrie 1970b,c) are that, at the die,

$$a = a_0 \quad \text{at} \quad z = 0 \tag{13}$$

$$u = u_0 \quad \text{at} \quad z = 0 \tag{14}$$

and, at the frost line,

$$\theta = 0 \quad \text{at} \quad z = z_1 \tag{15}$$

where z_1 is the frost line height (FLH). These have been discusssed in detail elsewhere and it will suffice here to identify a_0 with the die radius, to obtain u_0 from the (presumed given) die gap (h_0), density (ρ_0) at the temperature (T_0) of the melt emerging from the die and mass flowrate (M) using Equation (7), and to recall that the physical significance of (15) is that the force applied to the melt to draw it in the machine direction is applied axially and not in any other direction. We may note finally that a_0 and h_0 should be the dimensions of the emerging melt, rather than the actual die dimensions; that is to say, we should incorporate where possible a correction for die swell.

Now it would be perfectly reasonable to leave the problem at this stage, but the practical man may have a problem in using the model since it involves values of a force F and a pressure Δ which he does not normally know, and indeed it would be very difficult to measure F. If we take a hint from the way the process is actually run, we see that the next stage is to prescribe values of the blow ratio (BUR) and machine direction drawdown (MDD) defined by

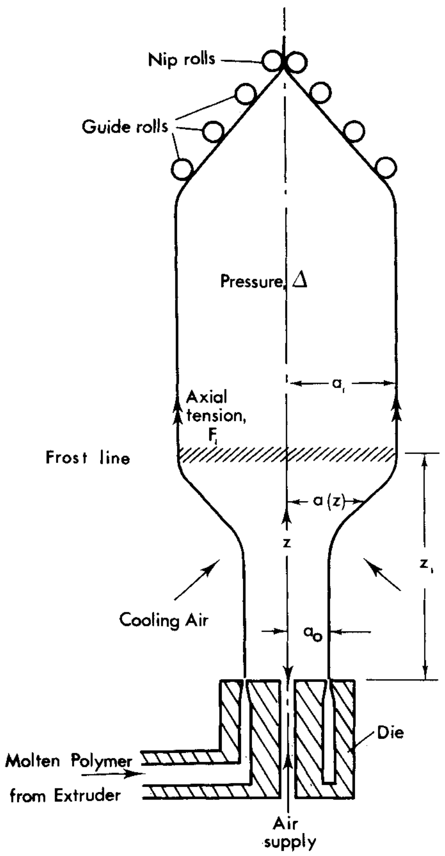

Fig. 1. Schematic diagram of the blown film process.

$$\text{BUR} = a_1/a_0 \tag{16}$$

$$\text{MDD} = u_1/u_0 \tag{17}$$

Then we seek values of F and Δ which give us the desired values of these ratios. This has been done so far by a simple-minded safeguarded linear interpolation which seems adequate and is efficient enough for this stage of the work. An apparent advantage of this approach is that we can reformulate the problem as an initial value problem and integrate numerically with starting values $a = a_1$, $u = u_1$, $\theta = 0$ at $z = z_1$. The iterations then alter F and Δ until the numerical estimates of a and u take their specified values, a_0 and u_0 at $z = 0$.

Effect of Gravity and Inertia

If we take into account gravity and inertia we obtain instead of (11) and (12), the three equations

$$\frac{2M\mu}{\rho}\frac{du}{dz} = -\frac{M\mu u}{\rho}\left(\frac{1}{a}\frac{da}{dz} + \frac{1}{\rho}\frac{d\rho}{dT}\frac{dT}{dz}\right) + \frac{u}{2}(F + \pi a^2\Delta)\sec^2\theta + \frac{Mu^2}{2}\sec\theta \tag{18}$$

$$2a(F + \pi a^2\Delta)\frac{d\theta}{dz} = \frac{2M\mu\cos^2\theta}{\rho}\left(\frac{3}{a}\frac{da}{dz} + \frac{1}{\rho}\frac{d\rho}{dT}\frac{dT}{dz}\right) - \frac{2Mga\sin\theta}{u} + (F - 3\pi a^2\Delta) - Mu\cos\theta \tag{19}$$

$$\frac{dF}{dz} = \frac{Mg\sec\theta}{u} \tag{20}$$

Instead of a value of a constant F, our iterative process now involves the initial value F_1 of the variable F, and the system is now fourth order, but otherwise we proceed as before.

The only difference of note is that for relatively large values of the dimensionless weight parameter

$$W = \frac{\rho_0 g a_0^2}{2\mu_0 u_0} \tag{21}$$

it becomes more difficult to find solutions because the iterative procedure fails unless the initial guesses for F and Δ are quite good. It seems possible that there are no solutions (to the problem as formulated here) for large enough values of W when, in physical terms, the film's weight makes it sag too much. This aspect of the problem has not been explored in great detail since it does not occur for realistic values of W in the cases discussed below, but two remarks may be worth making. W clearly goes up with a_0 as a_0^2 if u_0 is fixed, and linearly even if u_0 is increased proportionately—so the effect of gravity may be expected to be greater on larger units. Secondly, the computational problem is associated with the vanishing of $F + \pi a^2\Delta$ and the consequent breakdown of the numerical solution, something which may perhaps be avoided by including surface tension in the model (Petrie, 1972).

The magnitude of all these effects on computed results may be summarized briefly as follows. The effects of inertia and of density variation with temperature are extremely small. The effect of gravity is small but significant on bubble shape, and rather larger on the velocity profile. The variation of viscosity with temperature has the greatest effect and is the main factor contributing to the progress of the theory from predictions of rather unlikely bubble shapes (generally with very long necks and a very sudden blowing up just before the frost line) to predictions which are quite plausible, and in some cases close to experimental results.

Elastic Film

The use of a viscoelastic (Maxwell) model for melt rheology seems to be the next logical step, and the model has been set up formally (Petrie, 1973). However, there are considerable (numerical) difficulties in obtaining solutions of the boundary value problem we obtain, and as a preliminary to a fuller study the extreme case of a purely elastic melt has been considered. The equations may be obtained formally from those for the Maxwell model by letting the relaxation time (λ) tend to infinity (an infinitely long memory) while keeping the modulus ($G = \mu/\lambda$) finite. The equations can be integrated to reduce the order of the system from five to two. A direct derivation of the appropriate equations is, of course, quite straightforward, and provided the right choice of finite strain measure is made, the results obtained are identical.

The appropriate measure of strain is the Finger* measure, l^2/l_*^2 for an element of natural length l_*, and the constitutive relation for the extra-stress tensor is

$$p'_{ij} = G(C_{ij}^{-1} - \delta_{ij}) \tag{22}$$

In our case the strain tensor is diagonal, with elements

$$C_{ij}^{-1} = \text{diag}(u^2/u_*^2, a^2/a_*^2, h^2/h_*^2) \tag{23}$$

where the film has natural (unstressed) dimensions h_* and a_* at temperature T_0, and $u_* = M/2\pi\rho_0 a_* h_*$. We obtain for the nonzero principal stresses

$$L = G(u^2/u_*^2 - h^2/h_*^2) \tag{24}$$

$$C = G(a^2/a_*^2 - h^2/h_*^2) \tag{25}$$

and can eliminate h, as before, and substitute these equations into Equations (1) and (2). In this case (1) gives an algebraic equation relating u, θ, and a, which we may use to find u once we solve the second-order system of differential equations obtained from (2) and (3). For simplicity we shall ignore the effect of temperature variation. This will not be as serious an approximation as it is in the viscous case since the modulus (of a melt) is much less strongly dependent on temperature than the viscosity. In fact for LDPE the modulus is not affected by temperature according to Cogswell and Lamb (1970). Then the equations we get are

$$G\left(\frac{u^2}{u_*^2} - \frac{a_*^2\, u_*^2}{a^2\, u^2}\right) = \frac{\rho\, u \sec\theta}{M}(F + \pi a^2 \Delta) \tag{26}$$

$$a(F + \pi a^2\Delta)\frac{d\theta}{dz} = -\,2\pi a^2\Delta + \frac{MG\cos\theta}{\rho u}\left(\frac{a^2}{a_*^2} - \frac{a_*^2 u_*^2}{a^2\, u^2}\right) \tag{27}$$

We now have a slightly different situation from that for the Newtonian liquid, having a system of differential equations of order two instead of three, and also having two extra parameters, a_* and u_* (or h_*). A little thought shows that if we consider, as before, determining F and Δ to give us a known blow ratio and drawdown, Equation (26) used at the frost line gives a relation between a_* and u_* (taking $\theta = 0$ as before). Limited computational experience suggests two things—firstly that the value a_* chosen (taking this as the arbitrary parameter) has little effect on the bubble shape and velocity profiles computed; secondly, that solutions may not exist (and certainly have so far escaped detection) for some values of BUR, MDD, and FLH that are discussed below, indeed perhaps not for other than small values of MDD for realistic BUR and FLH values.

Apart from this problem, which may repay further investigation, it looks as though the best way to approach the problem for the Maxwell model is as a perturbation to the elastic model. We should use the equations already derived (Petrie, 1973) with G finite and $1/\lambda$ small and obtain corrections of order $1/\lambda$ to the elastic solution. This can be expressed as a perturbation in material parameters or in flow variables—the dimensionless quantity which has to be small is $u_0/a_0\lambda$ so we are thinking of the limit of high velocity or, with die dimensions a_0 and h_0 fixed, high throughput. The alternative of a perturbation to the Newtonian model is less appealing since we have a singular perturbation problem so that in terms of expected return on effort expended, resolution of the numerical problems inherent in the solution of the full Maxwell equations seems more attractive.

* The Finger strain measure corresponds to the Maxwell model expressed in contravariant tensor components with respect to a metric deforming with the material, and also to the Oldroyd fluid B. The author is grateful to Prof. A. S. Lodge for pointing out the incompleteness and inaccuracy of the description in the original manuscript.

COMPARISON OF COMPUTED AND EXPERIMENTAL RESULTS

In comparing solutions of the differential equations obtained above with experimental results, it is convenient to use dimensionless variables. The information needed from each experimental run is the BUR and MDD (already dimensionless), the FLH (divided by die radius, a_0), and measured temperature profile T (as a function of z/a_0). If gravity and inertia are considered the parameters W [Equation (21)] and

$$A = \frac{\rho_0 a_0 u_0}{2\mu_0} \tag{28}$$

are required (or for the elastic model similar parameters with G replacing $\mu_0 u_0/a_0$ as the characteristic stress). Then the iteration scheme produces values of dimensionless parameters

$$B = \frac{\pi\rho_0 a_0^3\,\Delta}{\mu_0 M} \tag{29}$$

and

$$H = \frac{\rho_0 a_0 F}{\mu_0 M} \tag{30}$$

The FLH, z_1, is taken to be the distance from the die at which the bubble radius is observed to become constant, which is slightly longer than the distance at which the temperature becomes constant. For simplicity, the observed temperature profiles were approximated roughly by two straight lines (or sometimes by a single straight line). An idea of the effect of this approximation is obtained by comparing predicted bubble shapes with two such temperature profiles which lie above and below the observed profile, and Figure 7 shows the results of this for one bubble. The temperature dependence of μ and ρ are taken to be

$$\mu = \mu_*\, e^{-b(T-T_*)} \tag{31}$$

$$\rho = \rho_*/(1 + c(T - T_*)) \tag{32}$$

Typical values of the constants for LDPE in these relations are $b = 0.03°C^{-1}$, $c = 0.00069°C^{-1}$, $\rho_* = 801$ Kg m^{-3} for $T_* = 115°C$. The actual value of μ_* is not required until we wish to calculate the actual pressure difference across film. Alternatively we can use measured values of

Δ and the computed values of B to calculate (see Equation (33), below) values of μ_* which we may compare with those obtained by other means. This apparent viscosity for what is an unequal biaxial elongational flow is more likely to be related to measured values of elongational viscosities, that is, it might be expected to be one-sixth of a measured biaxial elongational viscosity or one-third of a Trouton viscosity (see, for example Dealy, 1971).

Results of W. Ast

The results which Ast (1973) has published give bubble shapes for five runs which have the same mass flow rate and MDD, almost the same BUR and different FLH. We may note in passing that if the cooling air flow rate is altered to give a different FLH, the BUR is also affected unless the amount of air inside the bubble is adjusted—the process normally runs with a fixed mass of air inside the bubble. For one of his runs Ast also presents a velocity profile (u as a function of z).

In Figure 2 are shown the experimentally measured bubble shapes for two of these runs, compared with the predictions of the nonisothermal Newtonian model. The effect of the film's weight is shown by the results for $W =$

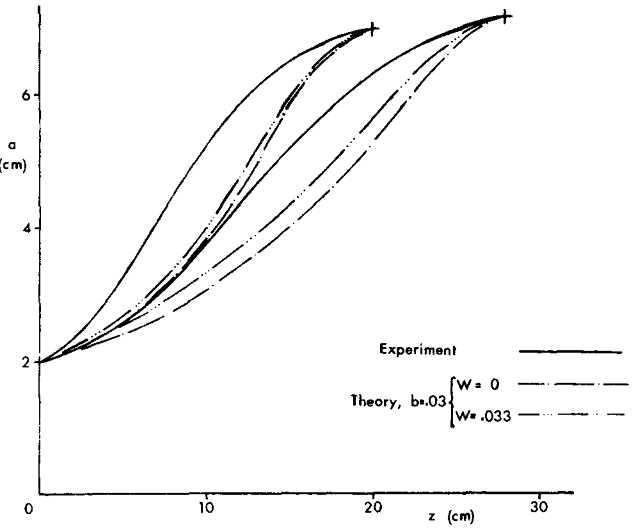

Fig. 2. Newtonian bubble shape predictions compared with two of Ast's measured shapes.

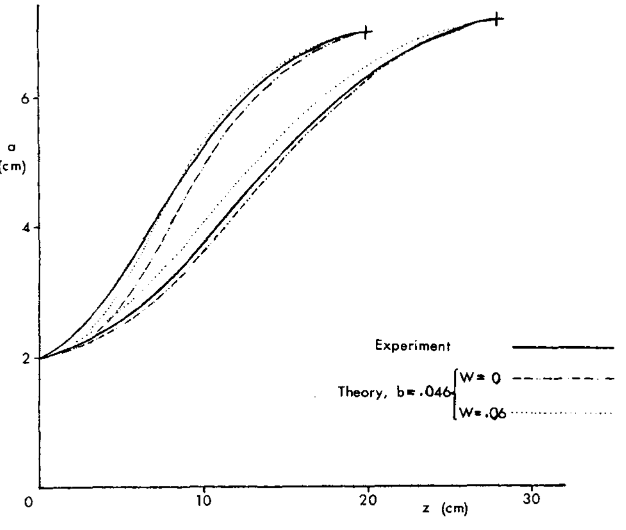

Fig. 3. Artificially improved Newtonian bubble shape predictions compared with Ast's results.

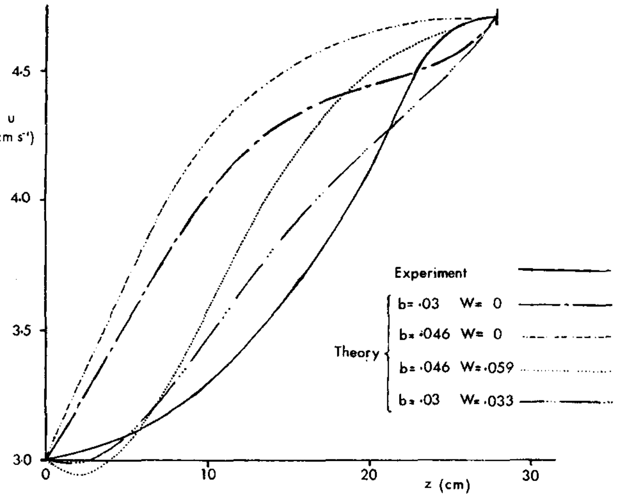

Fig. 4. Newtonian velocity profile predictions compared with Ast's results for two bubbles.

0.033 and while the change is towards better agreement, the magnitude of the effect is rather small. In fact the value of 0.033 for W is almost certainly an overestimate, so we conclude that gravity is unable to account for all the discrepancy between theory and experiment.

Figure 3 shows the unscientific way to get good agreement between theory and practice. We note that the most significant parameter in affecting bubble shape (apart from BUR and FLH) is the exponent b in the dependence of viscosity on temperature (31). We have used a value of 0.03, based on data supplied by F. N. Cogswell and by H. Münstedt. If instead we seek a value of b which gives better bubble shape prediction, we find that 0.046 does quite well. The dangers of this unethical approach are suggested first in the fact that the ideal value of b is different for each of the set of five runs (with the same polymer under very similar conditions). The temptation to follow this approach any further is removed when we look at Figure 4.

Here we see that the velocity profile predictions are considerably worse for the false value of b. We also notice that here the effect of gravity is quite large and goes a good way towards giving an acceptable prediction of the observed results. We must remember that the value $W = 0.033$ is high, however, so that the most honest estimate the theory gives lies somewhere between the two curves for $b = 0.03$.

When we come to consider the (isothermal) elastic model, we find that the bubble shape predictions lie to the other side of the measured values (Figure 5). This does seem very satisfying since we believe that the properties of the melt lie somewhere between the two extremes we have modeled but is probably no more than a happy accident. The velocity profile predictions are quite similar to those of the Newtonian model with gravity and are shown in Figure 6. Note that for the velocity profile the predictions of the elastic and viscous models do not bracket the experimental observations. As was remarked above, the choice of the arbitrary natural radius of the bubble does not affect these predictions, at least over a range from $a_* = 0.33\ a_0$ to $a_* = 1.24\ a_0$. However, the choice of a_* does affect the stress levels predicted so we cannot predict the bubble pressure from the modulus without making some assumption about a_*.

With an assumption that $a_* = a_0$, the approximate value of the dimensionless pressure difference predicted is, for the shortest bubble,

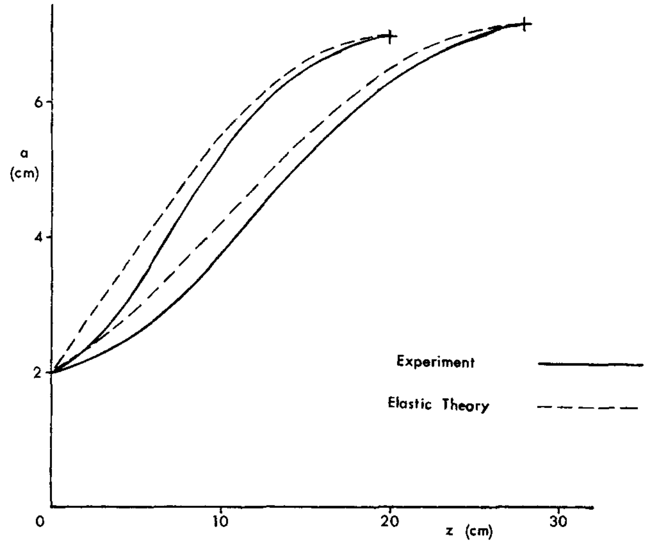

Fig. 5. Elastic bubble shape predictions compared with Ast's results for two bubbles.

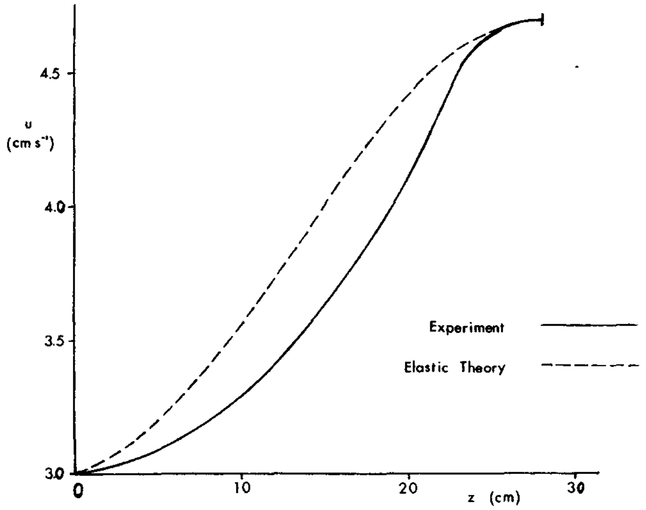

Fig. 6. Elastic velocity profile predictions compared with Ast's results.

$$\frac{a_0 \Delta}{2h_0 G} = 0.43$$

so with a modulus of 10^4 Nm^{-2} and $a_0 = 0.02$ m, $h_0 = 0.0006$ m, we have a predicted pressure difference of

$$\Delta = 260 \text{ Nm}^{-2}$$

From the Newtonian results if we take a value of $\mu_0 = 4 \times 10^4 \text{Nsm}^{-2}$ for $T_0 = 210°\text{C}$, we calculate, from the value found in the calculation for $b = 0.03$,

$$\frac{\pi \rho_0 a_0^3 \Delta}{\mu_0 M} = 0.24$$

using

$$M = 0.0017 \text{ Kg s}^{-1}, \quad \text{that}$$

$$\Delta = 860 \text{ Nm}^{-2}$$

Ast (1974) reports a range of 160 to 270 Nm^{-2} for the bubble pressure with this polymer so that the elastic model gives a reasonable prediction. The viscous model would give an acceptable prediction if the viscosity were about one quarter of the measured viscosity obtained in simple shear (at very low shear rates).

Results of R. Farber

The bubble shape predictions are again in reasonably good agreement with experimental results, as shown in Figure 7 for three of Farber's runs (Farber, 1973; Dealy and Farber, 1974). Here for simplicity a linear temperature profile has been used, using only the temperature of the melt as it leaves the die T_0 and its temperature at the frost line T_1. The viscosity data given by Farber fit Equation (31) with $b = 0.026$, and from results at low shear rate we obtain a zero-shear-rate viscosity of 1.23×10^4 Nsm^{-2} at 185°C. This is used to calculate W [Equation (21)] and A [Equation (28)] and after we find a solution and its corresponding values of B and H it is also used to calculate the pressure difference Δ and the axial force F. Results are shown in Table 1 for the nine experimental runs reported in Farber's thesis. These are compared with measured values of Δ, and in addition values of an apparent viscosity (μ_{app}) are shown where

$$\mu_{app} = \frac{\pi \rho_0 a_0^3 \Delta}{BM} \tag{33}$$

M, ρ_0, a_0, and Δ are all experimentally measured and B is obtained from the computer program. This value μ_{app} is an apparent viscosity in biaxial extension at the die temperature, 185°C. It may be of value if all our other assumptions are reasonable, but we should compare computed and experimental results further before coming to a definite conclusion.

Figure 8 shows graphs of strain rates in the machine and transverse directions against time for two of Farber's runs and the corresponding computed predictions. First this confirms that agreement is not so good for the shorter bubble; we may note that the transit time (from die to frost line) for the short bubble is 3.8s compared with a melt relaxation time (estimated from Farber's stress relaxation results) of 2s to 6s, depending on temperature. Thus we may expect elastic effects to be significant for the shorter bubble. As additional evidence of this we may note the comparison between observed and predicted transit times in Table 1; the agreement becomes better at larger transit times when we expect viscous effects to become more important.

Finally we may look in more detail at the strain rate graphs for the longer bubble and note that the main disagreement is near the frost line. As the melt freezes, experimental observation tells us that the strain rates all go

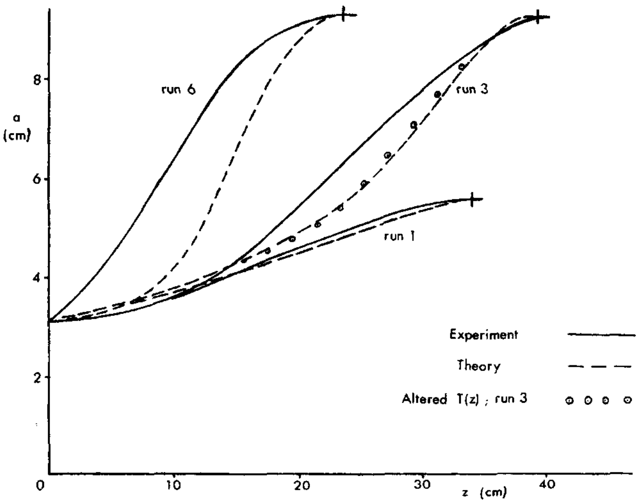

Fig. 7. Newtonian bubble shape predictions compared with Farber's results for three bubbles.

TABLE 1. COMPARISON OF THEORY AND EXPERIMENT USING FARBER'S RESULTS ($a_0 = 31.75$ mm, $h_0 = 1.27$ mm, $\mu_0 = 1.23 \times 10^4$ Nsm^{-2} AT 185°C)

Run	FLH (mm)	BUR	MDD	M(gs^{-1})	Δ (Nm^{-2}) Theory	Δ (Nm^{-2}) Expt.	Transit time, s Theory	Transit time, s Expt.	μ_{app} (Nsm^{-2}), equation (33)
1	340	1.76	6.75	2.18	192	46	9.5	7.8	2.94×10^3
2	276	3.44	3.56	3.72	284	66	7.4	6.3	2.86×10^3
3	392	2.94	4.16	3.07	195	51	10.5	10.7	3.22×10^3
4	427	2.55	4.94	3.04	192	51	10.3	9.6	3.27×10^3
5	457	3.30	6.99	2.77	183	56	10.5	7.3	3.77×10^3
6	236	2.95	7.03	2.74	312	53	5.8	3.8	2.07×10^3
7	220	2.75	6.88	2.14	256	66	6.9	4.2	3.17×10^3
8	225	2.30	12.06	1.54	233	56	7.0	3.0	2.95×10^3
9	254	3.00	9.24	2.42	289	53	6.1	3.0	2.26×10^3

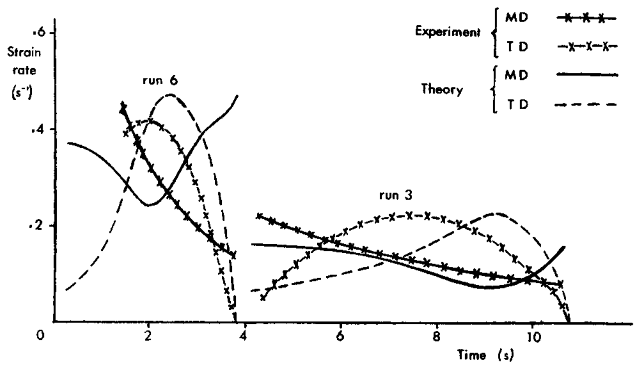

Fig. 8. Newtonian strain rate history predictions compared with Farber's results for two bubbles.

to zero, while in our model only the transverse direction strain rate is specified to be zero. We cannot specify any more in the mathematical model because there must be an axial viscous stress in the melt at the frost line to balance the applied force F. This is discussed in connection with the choice of boundary conditions by Pearson and Petrie (1970b). The basic discrepancy between our mathematical model and the real world is that the melt viscosity effectively becomes infinite when the polymer solidifies, and in Equation (31) we do not even have a very large increase in viscosity near the frost line temperature. There is evidence (Johnson, et al., 1968) that a better correlation than (31) would be a linear relation between log μ and $1/(T + 273)$, and these authors do report that the linear relation holds right down to the temperature at which freezing commences. We may note also that a temperature of 82°C at the frost line suggests that the melt (LDPE) is supercooled by perhaps 30°C.

Two numerical experiments should be reported in this connection. First the effect of the temperature profile specified was investigated by using two profiles which were upper and lower bounds to the experimentally measured profile. The result was a small change in bubble shape; the greatest effect is shown in Figure 7 for the longer bubble. In addition, the prediction of the pressure Δ changed significantly—for run 3 the range of predictions was from 156 Nm^{-2} to 204 Nm^{-2}, so our value of μ_{app} could be 25% higher. The order of magnitude of this change is not sufficient to bring the value of μ_{app} close to the measured shear viscosity μ_0. The second experiment was to alter the function $\mu(T)$ for T close to T_1 to make the melt behavior more realistic. It was possible to make the machine direction strain rate effectively zero at the frost line by this device, but the overall effect on the bubble shape and strain rate graphs was localized and depended on the way $\mu(T)$ was modified. This seems the best way to get good predictions from the Newtonian model, but it is likely that an ad hoc choice of viscosity function would be of very limited use, and it is by no means clear that the magnitude of the change in viscosity reported by Johnson et al. (1968) is enough to have the desired effect.

The obvious gap in the results presented is the absence of predictions for Farber's results using the elastic model. This is not entirely due to lack of time—so far it has proved impossible to find solutions of the elastic equations which give the required values of MDD, BUR, and FLH and it may be that there are no solutions for the larger values of MDD which Farber used (except with quite unrealistic values of BUR or FLH). There is no mathematical reason for supposing that the solutions we seek do exist and the failure may merely reflect a shortcoming of the elastic model. This problem does deserve further investigation, and it may be of value to compare the results of similar computations for melt spinning (Denn et al., 1974). It may be deduced from this work that there is no solution for spinning an elastic liquid as described by Equation (22) except with a draw ratio of 1, and indeed one of Denn's results is that there is a limiting draw ratio (a draw ratio which cannot be exceeded however great a take-up force is applied) for a Maxwell model also. The similarity between the two processes is sufficient to lead the author to expect similar limiting values of MDD in film blowing with a Maxwell model.

DISCUSSION

The mathematical model of the process has been subjected to a comparison with all currently available experimental data on blown film. Two questions which may be asked are "Is the agreement between theory and experiment good enough?" and "What use is the model anyway?" The latter question has been addressed in the section on conclusions and significance where the model is viewed as a necessary step towards a better understanding of the process which will help in design, prediction of product properties, and prediction of process limitation. On the whole, the predictions of bubble shape are reasonably good, and while there are a number of improvements to the model which might be expected to help here, the effort would not really be worthwhile. However predictions of velocity profile, and to an even greater extent strain rate, are not good. The situation is that, unfortunately, the most sensitive test of the theory depends on the more difficult experimental observations. Some work of Han and Park (1974) suggests that in fact a variable viscosity ('power-law' model) could improve the theoretical prediction appreciably, but with the lack of relevant data on melt properties this requires either direct experiment of appropriate power-law index (as in Han and Park's work) or some highly questionable assumptions.

We may also note the poor agreement between the observed bubble pressures and those predicted on the basis of a measured viscosity at low rates of shear. The predictions would be considerably better if the viscosity used in the calculation were reduced by a factor of about four. This is demonstrated by the apparent viscosity shown in Table 1, which does not vary very widely over the nine sets of results. The shear viscosity data of Farber and Dealy (1974) shows a corresponding reduction with a strain rate of about 2 s^{-1}, which is somewhat higher than a mean strain rate* for most of the experiments—values range from 0.3 to 1.4 s^{-1}, and there is no discernible correlation between the mean strain rate* and the apparent viscosity. The conclusion is that the material is more rapidly shear-thinning in biaxial extension than in shear, while the expected result for unaxial extension would be the reverse, although no data is available for this polymer. Denson (1972) has given evidence of similar behavior for polyisobutylene.

This sort of conclusion may have some value in making relatively simple engineering calculations, but the author does not recommend it as a substitute for further attempts to understand the way the viscoelastic nature of the melt influences the dynamics of the process.

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NOTATION

a = bubble radius, m
A = dimensionless inertia parameter, Equation (28)
b = viscosity exponent, Equation (31)
B = dimensionless pressure difference, Equation (29)
c = density coefficient, Equation (32)
C = principal stress in transverse direction, Nm^{-2}
C_{ij}^{-1} = Finger strain tensor, Equation (23)
F = total axial force, N
g = acceleration due to gravity, ms^{-2}
G = elastic modulus, Nm^{-2}
h = film thickness, m
H = dimensionless axial force, Equation (30)
l = length of a material element, m
L = principal stress in machine direction, Nm^{-2}
M = mass flow rate of melt, Kg s^{-1}
p'_{ij} = extra-stress tensor, Equation (22)
T = temperature of melt, °C
u = velocity of melt, m s^{-1}
W = dimensionless weight parameter, Equation (21)
z = axial distance, m

Greek Letters

Γ = surface tension, N m^{-1}
δ_{ij} = Kronecker delta
Δ = pressure difference, N m^{-2}
θ = slope of bubble profile, Equation (3)
λ = relaxation time of melt, s^{-1}
μ = viscosity, N sm^{-2}
μ_{app} = apparent viscosity, Nsm^{-2}, (Equation (33)
ρ = density, Kg m^{-3}

Subscripts

0 = value at die
1 = value at frost line
* = reference value, natural value

Abbreviations

BUR = blow ratio, blow up ratio
FLH = frost line height
LDPE = low density polyethylene
MD = machine direction
MDD = machine direction drawdown
TD = transverse direction

* The mean strain rate used is the square root of the second invariant of a mean rate of strain tensor. The components of this tensor are the total logarithmic strains divided by the transit time (all experimentally measured quantities)—for example, $\log_e$ (BUR)/(Transit time) in the transverse direction, $\log_e$ (MDD)/(Transit time) in the machine direction.

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